4.7 Load stiffening

It is known that the frequencies of a structure are altered if a state of stress is present within the structure itself. To understand this it is sufficient to think of a guitar string: its frequency varies depending on how much it is stretched or loosened. In particular, frequencies will be higher if the stress state is traction. The same thing happens in the blades of a helicopter when they are subjected to centrifugal acceleration.

To illustrate this effect we take the bar represented in figure 4.12 through the finite element model. We perform the calculation of the natural frequencies both when no load acts on it and when a thermal contraction is applied to it, able to generate a uniaxial stress state equal to 400 MPa.

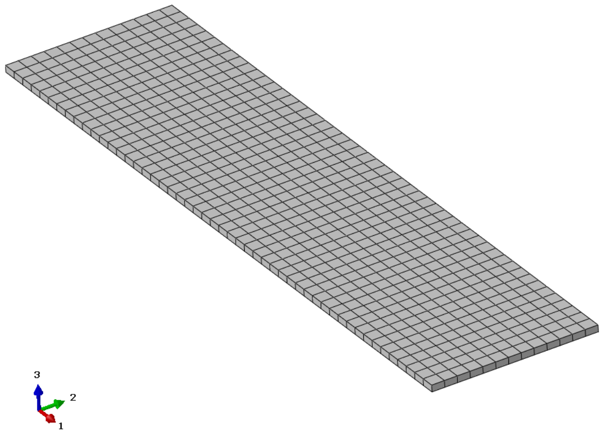

Figure 4.12. Model of a bar simply supported at the ends. Length = 200 mm, width = 50 mm, thickness = 2 mm. The model is made with shell elements and the ability of the pre-processor to plot the thickness of the elements was exploited.

Generally the calculation codes that allow the execution of modal analysis with load stiffening require that first the linear static analysis, for which the impact on the natural frequencies is to be evaluated, is carried out and then the modal analysis in the condition of the applied preload is executed.

Mode	Freq. (w/o preload)	Freq. (with preload)
1	1.168E+02	5.830E+02
2	4.706E+02	8.512E+02
3	6.384E+02	1.234E+03
4	1.068E+03	1.758E+03
5	1.342E+03	2.015E+03
6	1.913E+03	2.760E+03
7	2.168E+03	2.973E+03
8	3.009E+03	3.894E+03
9	3.160E+03	4.137E+03
10	4.348E+03	4.660E+03

Table 4.6. The presence of a stress state alters the natural frequencies of a structure, especially in the lower ones.

Table 4.6 shows the first ten natural frequencies of the bar under zero prestress conditions, along with the same information for the situation where a uniaxial stress state of 400 MPa (tensile) exists in the bar.

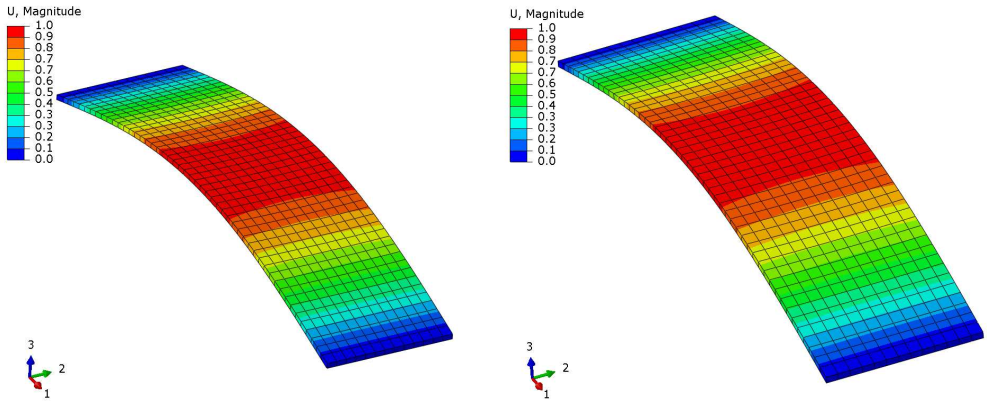

Figure 4.13. First mode shapes for the bar without preload (left) and with preload (right).

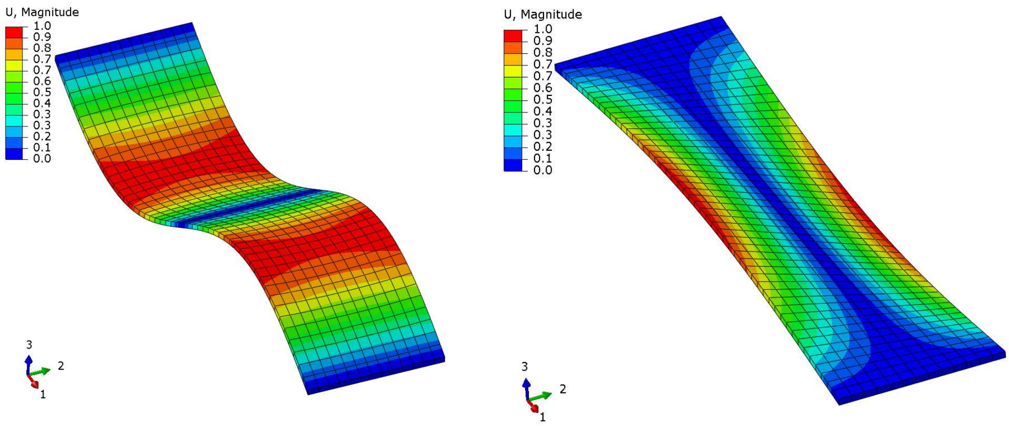

Figure 4.14. Second mode shape for the bar without preload (left) and with preload (right).

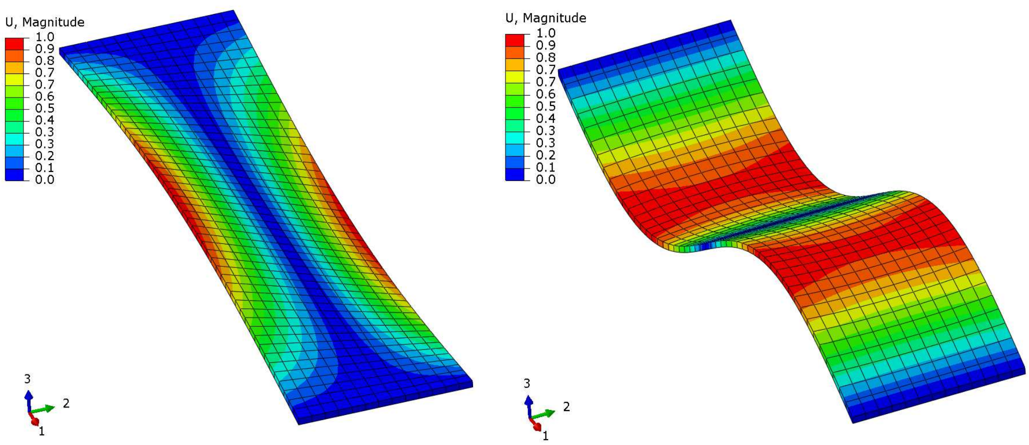

Figure 4.15. Third mode shape for the bar without preload (left) and with preload (right).

It is immediately apparent how the frequencies have actually increased. In addition, figures 4.13 through 4.15 also show the mode shapes of the first three modes. It is immediate to realize that the second mode reverses with the third.

Thus, a stress state not only alters the values of resonant frequencies, but can also shift and exchange some modes with each other.

The prerogative to consider preloads would be inherent in nonlinear codes; however, some purely linear codes make this capability available. From what has been said it is clear that a nonlinear code can analyze natural frequencies and mode shapes as a linear perturbation around the equilibrium position reached at any point in the loading history. But if there are contacts in the model, how does the code perform a modal analysis, which is only linear? Simple, it freezes the positions of the nodes at the point where they are, in fact "gluing" the contacts in the position they have at the equilibrium point and then performing the modal analysis.

4.8 Conclusions

In this Chapter we have barely touched on the dynamic aspect of calculating structures, and more details will be given later in the text. However, what we have seen here will be useful in Chapter 6, when we will discuss errors in finite element calculations.