designer will have to model all the details required for the construction of the element in question, even those that the structural engineer would not insert because of their lack of importance from the structural strength point of view.

Here then that, in case he had to handle a 3D model realized for constructive purposes, the experienced structural engineer would eliminate all the features that would make the finite element model uselessly complicated (in some cases the necessary interventions are of such entity that it is convenient to start from zero and build an ad hoc model), while the unprepared user, in doubt, would pass to the meshing phase without making any modification. This way of proceeding not only lengthens the time of solution, using more hardware resources, but can also create problems of numerical nature that affect the quality of the results. In fact the automatic meshers, when they have to deal with a complicated geometry, tend to create locally strongly distorted elements whose presence should be avoided because of the problems that we will illustrate in the following.

6.5.1 The condition number for the stiffness matrix

As already mentioned, the Finite Element Method assembles and solves a matrix equation of the type:

$$\{F\} = [K] \cdot \{u\} \tag{6.3}$$

where [K] is the global stiffness matrix of the structure, assembled by appropriately accounting for all the elements that model the structure itself.

To our purposes in the following examples we will mainly refer to structures consisting of a single element: the (6.3) is clearly still valid and [K] coincides with the element stiffness matrix.

One method of assessing the "numerical quality" of an element is to calculate the condition number of its stiffness matrix, defined as the ratio of the maximum and minimum eigenvalues. In other words:

$$C = \frac{\lambda_{max}}{\lambda_{min}} \tag{6.4}$$

The closer C is to unity, the better is the condition; a matrix [K] poorly conditioned is very "sensitive", in the sense that small changes in one or more of its coefficients create large variations in the results obtained with the (6.3), without altering the other factors. For example, let us have the following matrix relation:

$$\begin{bmatrix} 1 & -1 \\ -1 & 1.02 \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 4 \\ -2 \end{Bmatrix}$$

It presents the solution x = 104 and y = 100.

Let us now consider a system very similar to the previous one:

$$\begin{bmatrix} 1 & -1 \\ -1 & 1.01 \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 4 \\ -2 \end{Bmatrix}$$

The solution of this matrix equation is x = 204 and y = 200; in other words, a change of about 1% in one term of the coefficient matrix produces a change in the results of about 100%. This matrix is ill-conditioned and its condition number is, based on (6.4):

$$C = 202$$

6.5.2 Eigenvalues and eigenvectors of the stiffness matrix

Let's suppose we want to calculate the condition number for the stiffness matrix of the 4-node plane element shown in figure 6.60. Since the degrees of freedom of such an element are 8 the matrix will be an 8x8 and the "manual" calculation of the eigenvalues presents several difficulties.

We can proceed in two ways: the first one consists in requesting the calculation program to print on file the matrix and then calculate the eigenvalues by means of software able to perform such an operation [101][; the second way instead exploits the finite element code itself. In fact if we want to determine the eigenfrequencies of a structure the equation to solve is the following (considering null the structural damping - see Chapter 4):

$$[M] \cdot \{\ddot{u}\} + [K] \cdot \{u\} = 0 \qquad (6.5)$$

Where [M] is the mass matrix of the system.

Leaving aside the mathematical steps leading to the solution of the differential equation (6.5), we come to the following relationship:

$$\left| [K] - \omega^2 \cdot [M] \right| = 0 \qquad (6.6)$$

where ω are the eigenfrequencies of the structure expressed in radians per second.

By making [M] an identity (or unity) matrix [I] the (6.6) becomes similar to the

$$\left| [A] - \lambda \cdot [I] \right| = 0$$

used for the calculation of the eigenvalues λ of [A], with $\omega^2 = \lambda$.

Therefore, once performed a modal analysis for the element of figure 6.60 making sure that is [M] = [I] (the mass of the element must be equal to 4.0, so that each node