1.7 Axisymmetric stress

Axisymmetric stress status occurs in those bodies that present an axial symmetry, both in their geometry and in the forces that loads them (i.e. pressure vessels, rotating disks, hubs in general). These conditions make this stress status similar to plane stress or plane strain: in fact a point belonging to an axial symmetry plane can move, due to forces application, only in the plane which it belongs to.

In a cylindrical coordinate system where:

- let y be the axis coincident with the axial direction
- let x be the axis coincident with the radial direction
- let z be the axis coincident with the hoop direction

the stress strain relations assume the following form

$$\begin{aligned}
\sigma_{xx} &= 2\cdot G\cdot\frac{(1-\nu)\cdot\varepsilon_{xx}+\nu\cdot(\varepsilon_{yy}+\varepsilon_{zz})}{1-2\cdot\nu}\\
\sigma_{yy} &= 2\cdot G\cdot\frac{(1-\nu)\cdot\varepsilon_{yy}+\nu\cdot(\varepsilon_{xx}+\varepsilon_{zz})}{1-2\cdot\nu}\\
\sigma_{zz} &= 2\cdot G\cdot\frac{(1-\nu)\cdot\varepsilon_{zz}+\nu\cdot(\varepsilon_{yy}+\varepsilon_{xx})}{1-2\cdot\nu}\\
\tau_{xy} &= G\cdot\gamma_{xy}
\end{aligned} \qquad (1.17)$$

Note, by the way, that we did not use the classical notation r, θ, z which is in general applied for cylindrical coordinate systems; this has been done because calculation codes do not make any distinction. It is under the user responsibility to be aware of the fact that he is using the axisymmetric hypothesis (see also Chapter 2).

Using matricial notation:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xx}\\ \tau_{xy}\end{Bmatrix} = G\cdot\begin{bmatrix} 2\cdot\frac{1-\nu}{1-2\cdot\nu} & \frac{2\cdot\nu}{1-2\cdot\nu} & \frac{2\cdot\nu}{1-2\cdot\nu} & 0\\ \frac{2\cdot\nu}{1-2\cdot\nu} & 2\cdot\frac{1-\nu}{1-2\cdot\nu} & \frac{2\cdot\nu}{1-2\cdot\nu} & 0\\ \frac{2\cdot\nu}{1-2\cdot\nu} & \frac{2\cdot\nu}{1-2\cdot\nu} & 2\cdot\frac{1-\nu}{1-2\cdot\nu} & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\end{Bmatrix}$$

$$\{\sigma_a\}=[E_a]\cdot\{\varepsilon_a\} \qquad (1.18)$$

The “a” index indicates that we are referring to an axisymmetric stress status.

With respect to plane strain and plane stress, strain-displacement relations are somewhat modified and assume the following forms:

$$
\begin{aligned}
\varepsilon_{xx} &= \frac{\partial u_x}{\partial x} \\
\varepsilon_{yy} &= \frac{\partial u_y}{\partial y} \\
\varepsilon_{zz} &= \frac{u_x}{x} \\
\gamma_{xy} &= \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}
\end{aligned}
$$

And, using matricial notation:

$$
\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xx} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{1}{x} & 0 \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix} \cdot \begin{Bmatrix} u_x \\ u_y \end{Bmatrix}
$$

$$
\{\varepsilon_a\} = [D_a] \cdot \{u_a\} \qquad (1.19)
$$

On the other hand concerning indefinite equilibrium relations, equations (1.12) remain valid:

$$
[D_a]^T \{\sigma_a\} + \{f_a\} = 0 \qquad (1.20)
$$