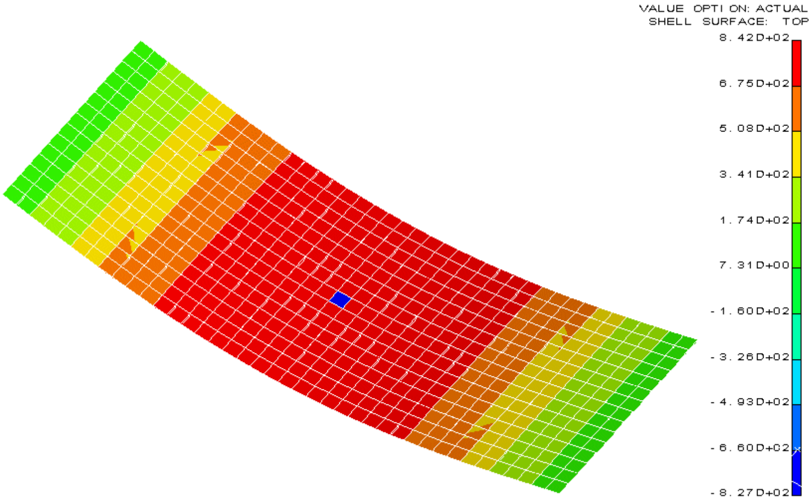

Figure 11. Non averaged stresses in longitudinal direction. It is possible to notice the discontinuity due to the presence of an up side down element.

4.4.2 Intersections among elements lying on different planes

Let us study another simple example considering the double T beam reported in figure 12 using its finite shell element model; in the same figure also the dimensions of the beam section are reported. The beam is clamped at one end and loaded at the other end by a vertical shear force F = 10000 N.

As the force lies in the web plane, the web will only see membrane stresses, while the two flanges will see both the membrane and bending stresses, even if the membrane ones will be the most relevant. In order to avoid dealing with local rising effects due to the discontinuity caused by the constraints, we will study the results in the section in the middle of the beam length.

Let us start with the stresses due to the bending effect, calculating their values in the locations illustrated in figure 13. We will begin using the Solid Mechanics relations and then we will use the finite element model.

$$\sigma_A = \frac{M_{fmid}}{I} \cdot \frac{h_A}{2} = 161.8 \text{ MPa}$$

$$\sigma_B = \sigma_C = \frac{M_{fmid}}{I} \cdot \frac{h_B}{2} = 152.1 \text{ MPa}$$

being:

$$M_{fmid} = F \cdot \frac{L}{2} = 2500000 \text{ Nmm}$$

$$I = 772366\ mm^4$$
$$h_A = 100\ mm$$
$$h_B = 94\ mm$$

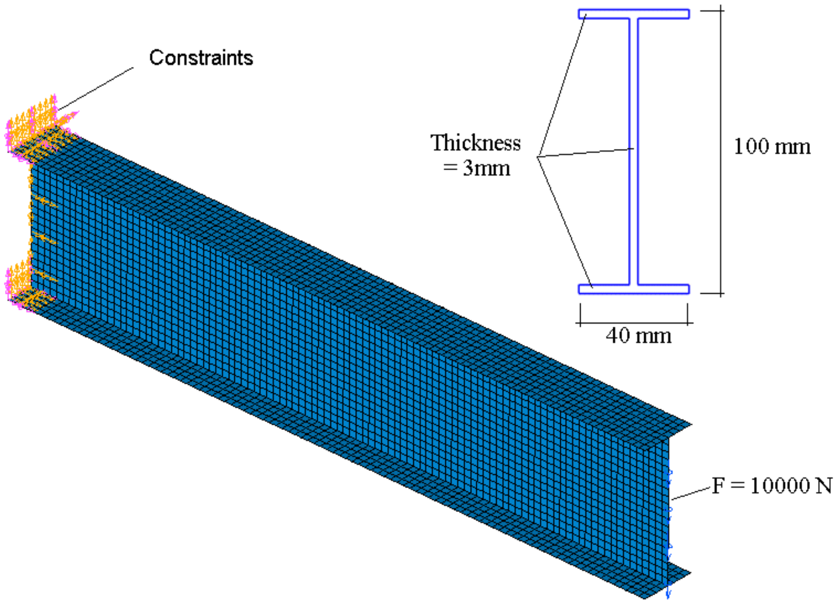

Figure 12. The beam has a length L = 500 mm. Shell elements lie in the middle planes of web and flanges.

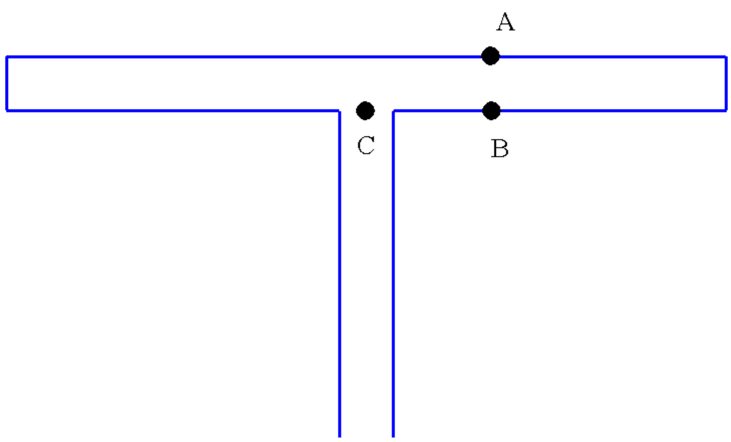

Figure 13. Points where the stresses are calculated.

In figure 14 the longitudinal stress contour is reported for the top faces that, in the case of the upper flange, is the value to be compared with σ_A, while figure 15 reports the same information but for the bottom faces, to be compared with σ_B and σ_C.

In both cases the error is very small, around 1÷2%.

We notice that while in the theoretical case σ_B and σ_C have the same value because the distance from the neutral axis is the same in both conditions, in the finite element model there is a difference, which is small but nevertheless not negligible. This happens because in point C of the model the stress, averaged, takes into account also the contribute of the element belonging to the web which intersects the flange. But why is the value higher? Looking at the numerical results (i.e. by clicking with the mouse on a non averaged plot) it can be noticed that in the elements of the flange close to the intersection with the web the same stress as σ_B is recorded, as shown in figure 16. Therefore, as the averaged value is higher, the responsible for this must be the element belonging to the web; in fact, looking at figure 16, it is possible to see that in this element the stress is 155.3 MPa. We remind that shell element models are generally built meshing the middle surfaces of the structures; in this example, then, the web has not an height equal to $(100 - 2 \cdot 3) = 94 \text{ mm}$ but equal to $(100 - 2 \cdot 1.5) = 97 \text{ mm}$. Therefore the model "catches" in the web a stress calculated as:

$$\sigma_{CFEM} = \frac{M_{fmid}}{I} \cdot \frac{97}{2} = 156.9 \text{ MPa}$$

with an error still around 1% with respect to the theoretical value, but aligned with what has been found for the other points far from the intersection with the web.

This effect is more evident as the thickness of the elements bigger becomes, even if in a model built with shells the thickness should not be to big due to their nature. In this simple case it is easy to understand that the correct value to be used is the one given by the flange elements, while sometimes in more complex models it could be more difficult.

The last consideration concerns the distinction between top and bottom for web elements; we said that the web works exclusively in a membrane status and therefore, as the bending components are not present, the stress for top, bottom and middle is the same.

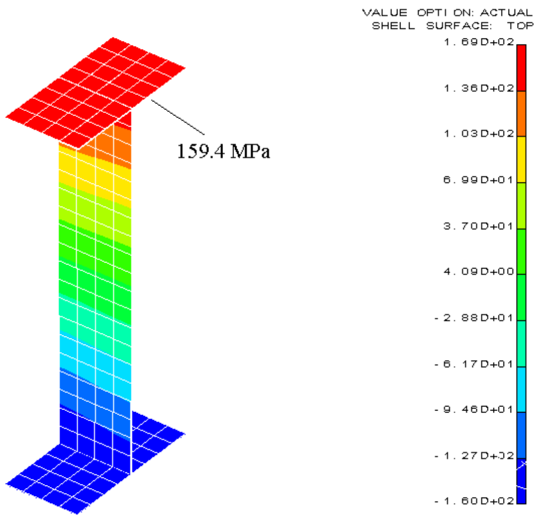

Figure 14. Longitudinal stress in the middle section of the beam: top faces.

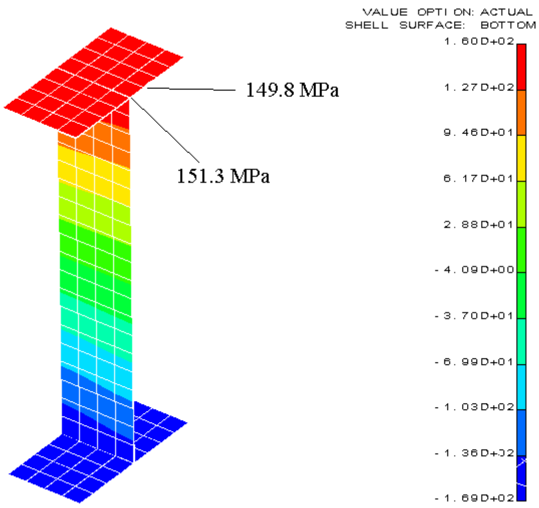

Figure 15. Longitudinal stress in the middle section of the beam: bottom faces.

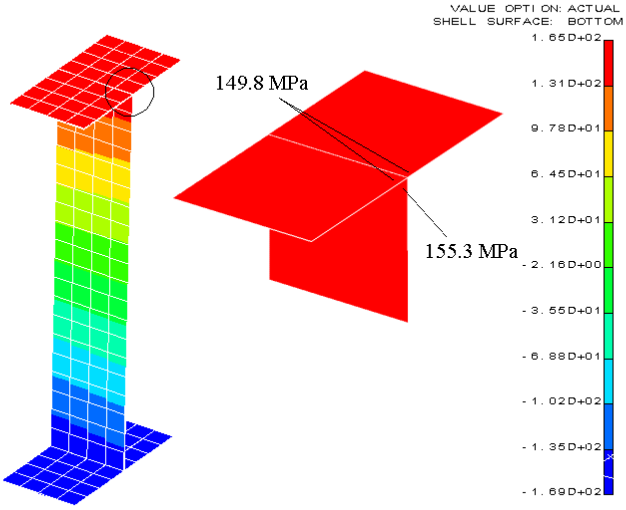

Figure 16. Non averaged longitudinal stress in the middle section of the beam: bottom faces. An enlarged portion of the model is reported in order to illustrate the stress values, in the interface node, for the web and flange elements.

Let us now consider the stresses related to the shear effects. The shear force is constant along the beam axis; for this reason we could analyse an arbitrary section. In order, as already discussed, to avoid any possible problem related to the constraints effects, we will analyse the section in the middle of the beam length.

Solid Mechanics tell us that the maximum shear stress is located in the centre of gravity line and its value is given by:

$$\tau_{max} = \frac{F \cdot S^*}{I \cdot b} = -39.4\,\text{MPa}$$

being $S^* = 9132.4\ \text{mm}^3$ the static moment, for the definition of which we send back to Solid Mechanics books, and b = 3 mm the thickness of the web.

Moreover we know that the τ in the flanges are not equal to zero and that they can be evaluated using the same relation above, substituting the actual value for S^* and b. As an example, in the flanges and at 15 mm from the edge, the value for the shear stress is ±9.4 MPa (the sign depends on which side of the web we are calculating the stress).

In figure 17 the shear stress contour is reported for the middle section of the beam. In the centre of gravity axis the value calculated by the model is -39.3 MPa, while in the flanges is ±9.2 MPa; in both location the values are practically coincident with the theoretical ones, as it can be noticed in figure 17.

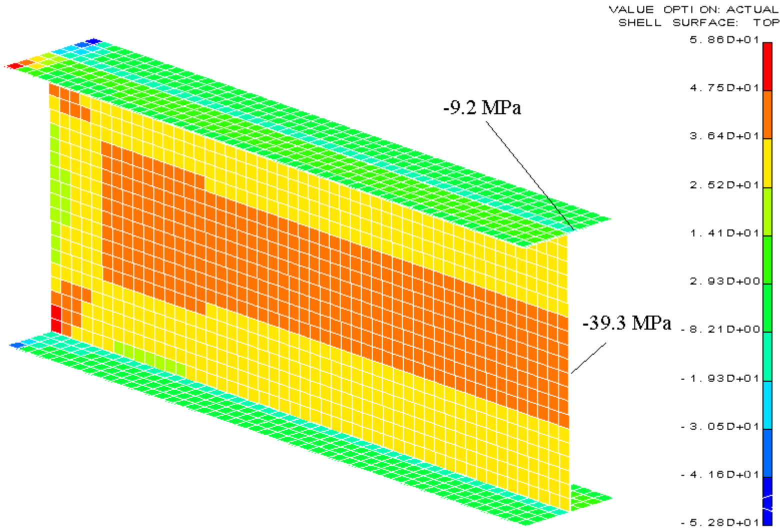

Figure 17. Shear stress contour. Both in the web and the flanges the agreement between theory and finite element model is very good

4.4.3 Discontinues junctions

In Chapter 2 we mentioned a few systems to join different parts of the same structure modeled with shell elements; in this paragraph we will deal with this matter when the joining system is based on discrete points. Let us consider the beam built with three plates joined together as shown in figure 18; the structure is loaded by a shear force F = 12500 N (and a bending moment also arises). Our task is to construct a junction among the parts using rivets or other systems that can transmit the loads mainly by shear actions. Following the classic approach it is necessary to make some assumptions on the number of the rivets and their diameter and then to check by calculations the correctness of the hypotheses. Let us suppose to use 5 rivets with a 10 mm diameter each, with a pitch among them equal to 20 mm, thus leaving 10 mm from the edges of the plates. We obtain the condition shown in figure 19.

As the junction line is exactly in the center of the beam, the bending moment acting in this section is given by:

$$M_f = F \cdot \frac{L}{2} = 12500 \cdot 200 = 2500000 \text{ Nmm}$$

We know that the bending moment gives a “butterfly” force distribution (like stresses) and therefore on the rivets we will have a situation like the one illustrated in figure 20.