APPENDIX A

The solution of linear algebraic equation systems

A.1 Introduction

When we have to face the numerical calculation of a structure using FEM it is necessary to establish the accuracy of the model that we are going to build; the precision level is in general a function of various parameters mutually related that can anyway be resumed in the following assertion: model complexity mainly depends on the phenomenon that is being investigated.

It can therefore occur, mainly thanks to the power of modern modelling software and to the birth of automatic meshers, to get to the creation of so huge models (despite the use of all the adequate systems to reduce their dimensions, as we have seen in Chapter 6, through the substructuring and submodelling techniques) that they can create problems during the solution phase, although we could have at our disposal very big hard disks where the software can write the necessary temporary files. As it is known, solution times and required hard disk space grow exponentially with the number of algebraic equations of the system that FEM builds on the information given by the structural engineer. Therefore, beside the power increase of computers, it is necessary to optimize the software that numerically solves the equation system.

Here we will just compare two numerical solution techniques for linear system and we will see their advantages and their disadvantages.

A.2 The equation system

However before proceeding in this direction it is necessary to briefly describe how the system that has to be solved, in order to know the linear elastic behaviour of a structure, is constituted.

FEM, through interpolations and relationships that we will not recall here but that we will see in Appendix B, assemble a linear system that in matricial form is generally written in the following way:

$$\{F\} = [K]\cdot\{u\} \tag{A.1}$$

where {F} is the vector of the active forces (known) and the reactive forces (unknown), {u} is the vector that contains the displacements (known in the constrained nodes, unknown elsewhere) of the nodes that discretize the structure and [K] is the so called stiffness matrix (see Appendix B) whose terms are completely known when the geometry of the structure and the material with which it is built are known. It must be noticed that in the nodes where the displacements are known the reaction forces are unknown while where the forces that load the structure are known the displacements are unknown; therefore by dividing into an adequate way the matricial equation (A.1) it is possible to achieve the determination of all the unknowns. If we want to calculate only the displacements and to shift to a following solution step the calculation of the reaction forces we can write the system (A.1) in an analogous form:

$$\left\{F^1\right\} = \left[K^1\right] \cdot \left\{u^1\right\} \qquad \text{(A.2)}$$

where the terms have the same meaning previously indicated but they are now referred to the subsystem obtained by reordering the terms in order to calculate only the displacements.

The solution of equation (A.2) is obtained by “simply” inverting matrix $[K^1]$, thus arriving to the following relationship:

$$\left\{u^1\right\} = \left[K^1\right]^{-1} \cdot \left\{F^1\right\} \qquad \text{(A.3)}$$

Anyone who knows just a bit of matrix algebra certainly knows that inverting a matrix bigger than a 3x3 one presents some difficulties from the point of view of pure calculations; it is then easy to imagine which kind of complications we could face having to solve a matrix that can easily reach order 100000 (even with codes that run on PC!).

Nevertheless $[K^1]$, for reasons that are strictly related to finite element modelling, has some characteristics that take it back to a more manageable form if compared to generic matrices:

- it is symmetric and therefore only a half can be stored (upper or lower triangles) plus the elements on the principal diagonal;
- it is “sparse”, i.e. it is essentially constituted by null elements, with some terms different from zero (besides the ones on the principal diagonal).

It is mainly this second property that facilitate the solution of equation (A.2), both if we use direct methods and if we use iterative type techniques.