although with a continuous model the method based on the fluxes is however possible. In any case, whether we are dealing with continuous or discontinuous seams, we will have to be in the situation, already seen in Chapter 3, shown in figure 10.5.

The discussion of spot welding basically falls into the case of rivets. With a suitable model (continuous with flux extraction or discontinuous with force extraction at the connection points), the forces acting on the single point are compared with the allowable values for the button. And precisely herein lies the biggest problem: the strength of the weld spot is highly dependent on the process by which it is made, and therefore it is necessary to have experimental data to be able to judge the integrity of the joint.

10.3 Fatigue assessment for homogeneous and isotropic materials

10.3.1 Continuous structure parts

Since the discovery of the material fatigue phenomenon dates back to quite recent times, still today there are several uncertainties in the calculation and determination of the fatigue strength of a given structure, even if it is made of homogeneous and isotropic material.

10.3.1.1 Classic Method

The classical method, i.e. based on hand calculations, proceeds as follows.

Consider the notched bar, already seen in Chapters 6 and 9, whose geometry we report here in figure 10.6.

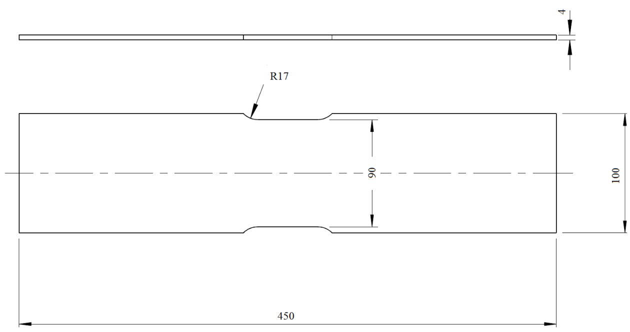

Figure 10.6. Notched bar subjected to an axial force F = 75000 N cycling from zero.

Let the load be a fatigue type axial force, cycling from zero to a peak of 75000 N. We know (see Chapter 6) that the stress in the smaller section is σ_2 = 208 MPa. Suppose that the material constituting this plate has a limit, on specimens without notches, at infinite life and for an alternating symmetrical load from zero (i.e., with cycles that change the stress from tensile to compressive), equal to:

$$\sigma_{FAb} = 255 \text{ MPa}$$

In order to be able to determine the allowable limit for our type of fatigue and for the notched geometry of the bar, it is necessary to construct the Goodman-Smith diagram, for the plotting of which we refer to Machine Design texts [11]. Figure 10.7 illustrates the diagram we obtain with the data in our hands:

$$K_t = 1.45 \qquad\qquad K_f = 1 + q \cdot (K_t - 1) = 1.38$$

having assumed the notch sensitivity factor q = 0.85 (q is the greater the higher the mechanical strength of the material or, better, the closer the yield and ultimate limits are to each other). We observe that, as long as q < 1, the fatigue notch coefficient K_f is less than K_t. Even if q = 0 (total insensitivity to notching: extremely ductile materials) we would have $K_f = 1$. Vice versa, if q = 1 (total sensitivity to notching: extremely brittle materials) we would have $K_f = K_t$. The fatigue notch coefficient is used, together with two other reduction factors, in the following way to "reduce" the experimental value σ_{FAf} obtained on unnotched specimens:

$$\sigma_{FA} = \frac{\sigma_{FAb} \cdot b_2 \cdot b_3}{K_f} = 149 \text{ MPa}$$

having assumed for b_2 (surface finishing factor) a value equal to 0.9 and for b_3 (dimensional factor) a value also equal to 0.9 (we will come back later on the coefficients b_2 and b_3).

The red line represents the ratio:

$$K = \frac{\sigma_{max}}{\sigma_{average}}$$

Since $\sigma_{max} = 2 \cdot \sigma_{average}$ we get K = 2.

The allowable value is obtained from the intersection of this line with the blue polygon.

From the Goodman-Smith diagram we can then obtain $\sigma_{lim} = 230$ MPa. With this value we can finally calculate the fatigue safety coefficient for the plate in discussion:

$$CS = \frac{\sigma_{lim}}{\sigma_2} = \frac{230}{208} = 1.1$$

In the "classical" method, therefore, the fatigue notch coefficient is used to lower the value of the allowable stress.

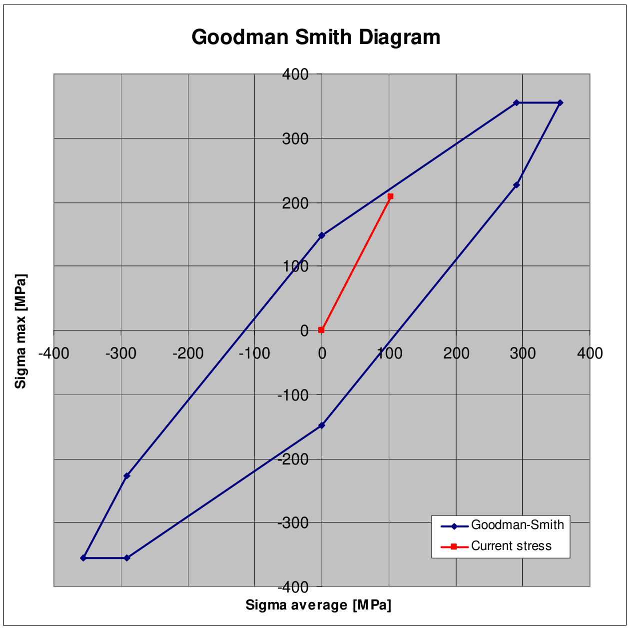

Figure 10.7. Goodman-Smith diagram of the material used for the bar. Information on rupture (σ_{Rt}) and yield (σ_{Yld}) is also required for plotting. The area enclosed by the blue line is the range within which the stress cycles must be contained in order to have a safety coefficient greater than or at the limit equal to 1. The red segment represents the current cycle and its intersection with the polygon in blue gives the limit value, equal to 230 MPa.

This in a finite element calculation is not possible, since, as we have seen, the method is able to catch the real stress distribution. Let us then see how fatigue verification can be done when the results are obtained from a finite element model. The only knock-down coefficient we take into account is the one related to the surface state (b_2); we will verify a posteriori if neglecting b_3 (or better having assumed it equal to 1.0) is legitimate or not. We will have:

$$\sigma_{FAFEM} = \sigma_{FAb} \cdot b_2 = 229 \text{ MPa}$$

With this value, we obtain the Goodman-Smith diagram shown in figure 10.8 and we derive:

$$\sigma_{\lim FEM} = 316 \text{ MPa}$$

In figure 10.9 we then report the value of the stress that is obtained from the model (see also Chapter 6). Ultimately, the following value of the safety coefficient is calculated:

$$CS = \frac{\sigma_{limFEM}}{\sigma_{FEM}} = \frac{316}{311} = 1.02$$

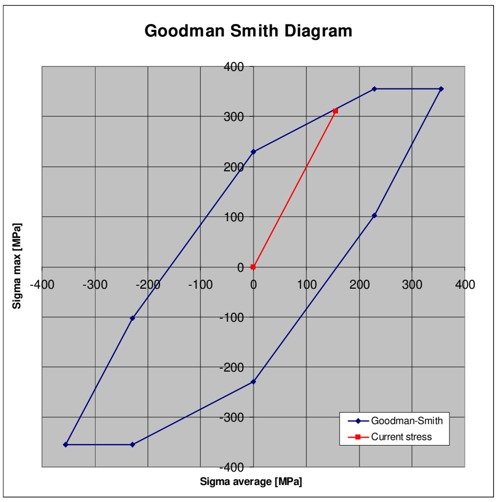

Figure 10.8. Goodman-Smith diagram plotted for the FEM calculation. Clearly σ_{Rt} and σ_{Yld} remained unchanged.

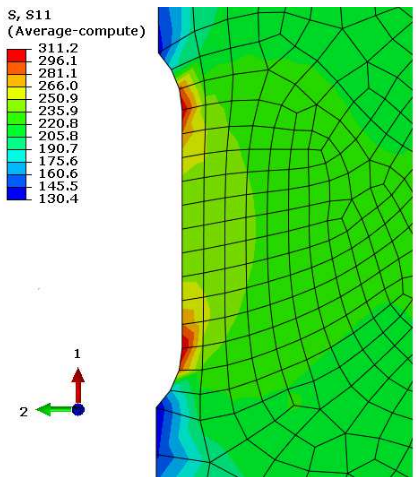

Figure 10.9. The maximum stress is σ_{FEM} = 311.2 MPa.

As we can see, the value of the safety coefficient obtained with the finite element model is lower than in the previous case, indicating that it is more conservative. This tendency is quite general and can be attributed to the fact that basically the notch sensitivity factor q is not considered, or rather it is assumed to be equal to 1. This discrepancy clearly increases the more ductile, and therefore the less sensitive to notching, are the materials used.

However we know, having calculated $K_t = K_f$, that in reality the stress to be considered for the fatigue calculation is lower than that determined by the model and therefore we could think of lowering the value of σ_{FEM} through the ratio $K_f / K_t = 0.95$, thus obtaining:

$$CS = \frac{\sigma_{limFEM}}{0.95 \cdot \sigma_{FEM}} = \frac{316}{295} = 1.07$$

that is a value more in line with what we have calculated by hand. We observe, however, that proceeding in this way would lose all the advantages of the Finite Element Method, since we would have to determine K_t every time. Since, proceeding in the way we have seen, we are in favor of safety, it is more convenient to follow the approach shown.

10.3.1.2 The Gough-Pollard criterion

In practice, then, it is seldom that we have to deal with a simple stress state such as the monoaxial stress state now seen; even if in the notch zone there are also other components of the stress tensor, the one examined remains essentially a monoaxial stress state. But how can we proceed when we have to treat a complex stress state? The Gough-Pollard criterion illustrates how to do when we have both a σ and a τ, as it happens in the case of transmission shafts in general: next to the σ stress generated by the bending we have also the shear stress τ produced by the torque. We could extend this criterion, which has a very good experimental confirmation, also to cases not strictly inherent to transmission shafts or axles (for example for screws that also work in shear), but if only we have a case in which, in addition to one σ and one τ, we also have another stress σ, clearly in the direction orthogonal to the first, we no longer know what to do; that is, if we are faced with a state of plane stress in which all three components σ_{xx}, σ_{yy} and τ_{xy} are non-zero, checking fatigue strength at that point seems difficult. Fortunately, this is the most complicated condition we can deal with, because in any case we know that, apart from particular situations such as bodies in contact, the greatest stress in an organ is on its surface, where the stress state is planar by definition (an exception is constituted by pressure vessels, where on the internal surface, in contact with the fluid, the stress state is triaxial).

The Gough-Pollard criterion calculates an equivalent σ_{GP} stress as follows:

$$\sigma_{GP} = \sqrt{\sigma_{max}^2 + H^2 \cdot \tau_{max}^2}$$

being $H = \frac{\sigma_{lim}}{\tau_{lim}}$; the two limit values are derived from the corresponding Goodman-Smith diagrams, i.e. it is necessary to create one for the shear stress as it is done with the σ.

The safety coefficient is then calculated as follows: