14.2.7 Conclusions

From what has been said in the preceding paragraphs, it is difficult to judge a priori which method is best to use, given the merits and demerits of each. However some general indications can be given.

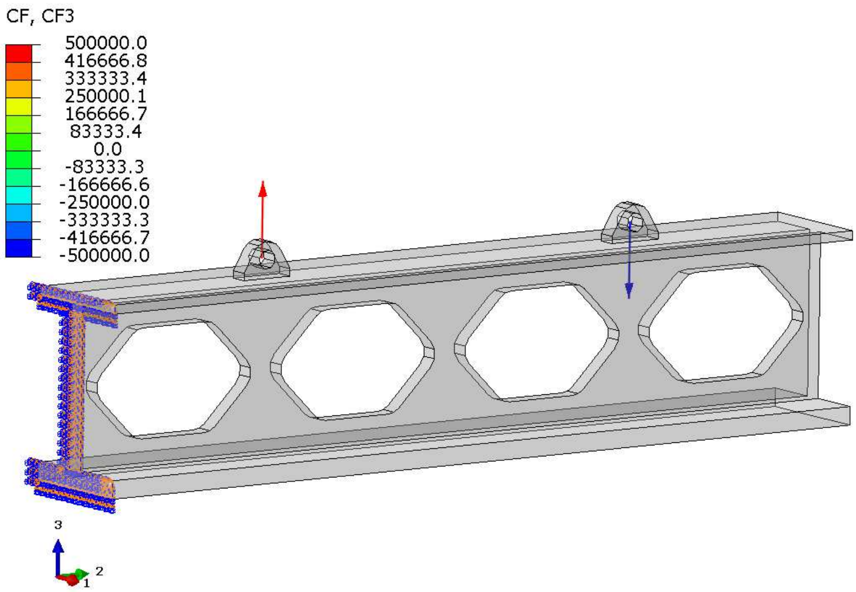
- If the model of the system to be analyzed is not too large it is better to use the method of direct integration, so it is possible to avoid the modal analysis, especially if several modes are expected within the range of frequencies to be analyzed.
- Vice versa if the model is of considerable size it is advisable to use the method of modal superposition; in this case remember to check the effective modal masses and to take a number of modes that gives an effective modal mass equal to at least 85% of the total mass (this applies to masses and moments of inertia), although just above we have seen that even with 81% we had a correct answer (assuming that the most correct solution is that given by direct integration); we underline however that this was a definitely simple case, with a single structure and loaded only in one direction.
- If the force has a precise frequency it is always better to use the direct integration method, thus avoiding the choice of the number of modes that should be involved in the modal superposition. Moreover, in this hypothesis, it would be even useless a modal analysis because the study of the frequency response is already sufficient.
- Finally, it must be said that the cases in which a structure works around a resonance are quite rare; in fact, generally we tend to make sure to be far enough from the frequencies of the system. There are, however, very particular cases, such as the joining of plastic parts by means of ultrasound, where an element, called "sonotrode", vibrates at its resonance frequency and, placed in contact with the joining area, locally melts the materials creating the weld.
- Apart from particular cases such as the one just mentioned, it could happen that in the transient, i.e. in that phase in which the force reaches its operating frequency (think of a centrifugal compressor that works at a very specific speed, but clearly will see the startup and shutdown), we will go to excite some frequencies away from that of operation, then it may be worthwhile to see what happens in these phases using the calculation of the response over time (time history).

14.3 Transient dynamic analysis

This is the most general case of dynamic response. The equation to solve is always the (14.1), but in this case $\{F(t)\}$ can vary in time, following a history that can also be of random type, as it happens for earthquakes and wind forces for civil structures or for forces induced by the irregularity of the road surface, as it happens for land vehicles (for these situations, however, different methods are used, which we will mention later). Also in this case we can use the two solution techniques seen previously for the harmonic analysis with the same considerations reported just above. The only difference, which is not so small, is that if it is necessary to take into account geometric, contact or material nonlinearities, the only method that can be used is the direct integration.

In order to be able to describe the force F as a function of time t, calculation codes generally provide a table-type system that allows the input to be described by points. Often this information comes from other computational codes (e.g. multibody, see Chapter 18) or from experimental measurements.

In this paragraph we want to highlight the reason why, sometimes, it may be convenient to perform a transient dynamic analysis instead of a classical static analysis with which too often we tend to oversimplify phenomena. In order to do this, let's consider again the beam used in this Chapter, but this time with only one end clamped to ground and the other free, as illustrated in in figure 14.12.



Figur3 14.12. Beam clamped at one end and loaded with two opposing forces.

Similarly to what we did in the previous paragraph, we apply to the rings two vertical and opposite sign forces equal to 0.5 MN according to the time history shown in figure 14.13. As we can see, the forces grow from zero to the maximum value in 1/100th of a second, remain applied for 1/100th of a second and then in the same time they cancel again: a fast time-varying phenomenon, then.

Since the constraint conditions have changed, the eigenfrequencies and mode shapes of the beam will also have changed. Therefore, if we wanted to follow the modal superposition approach we would have to perform a new modal analysis, check the effective modal masses and determine if the number of extracted modes is sufficient or if we need to repeat the calculation with more modes. We still follow both paths.

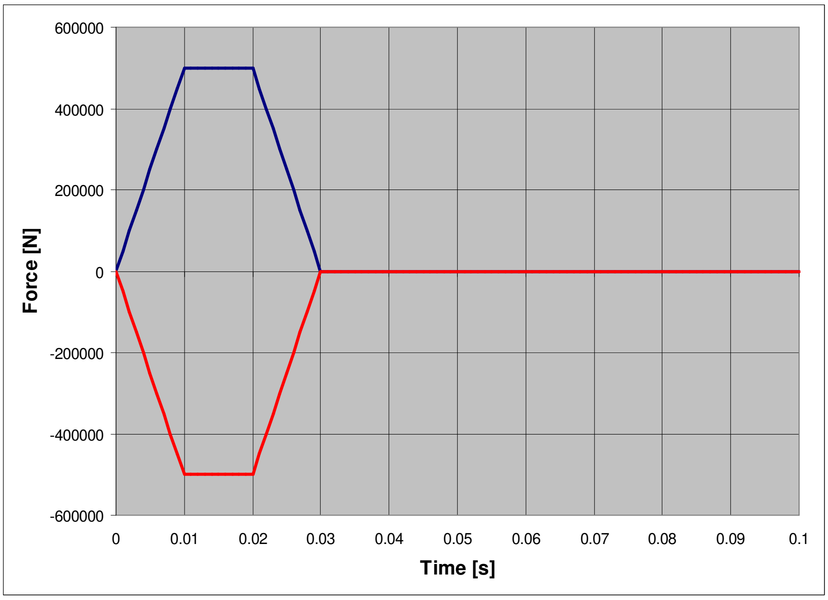


Figure 14.13. Time history of the forces applied to the beam.

14.3.1 Direct integration

The first problem we have is the value to be given to the time step used to integrate the (14.1) with numerical methods; a rule of thumb is to use a Δt equal to 1/100 of the total time for which we want to follow the phenomenon (we emphasize that here we refer only to the implicit method: for the explicit method there are in fact different rules, as we will see later). Wanting to see what happens to the system in 1/10 of a second, we will use a Δt equal to 0.001 s. It must be said then that, usually, the calculation codes adapt the step according to the speed of convergence of the solution, trying to increase it as much as possible to reduce the calculation time; but we must be careful, because some peaks that the code cannot realize could be "lost": for this reason it is good practice also to set a limit above which it is good that the program does not go.

Finally, if the time history of the force had a particularly "dynamic" trend, the time step must be such that it can follow it adequately. As an example, the history of figure 14.14 lasts approximately 0.5 s and therefore, in base to the cited empirical rule, a time step equal to 0.005 s would have to be fine, but we realize that with a step of this kind it would not be possible to follow the force in an adequate way.

In these cases, once the step to be used has been established, it would be advisable to carry out the calculation on a simplified model that, at the extreme limit, can only contain the nodes, bound to the ground, to which the force is applied and some point masses attached to those nodes; this is only to verify if, with the chosen step, it is possible to reproduce the time history of the force.

Our example is very simple and consequently a step of 0.001 s is more than adequate.

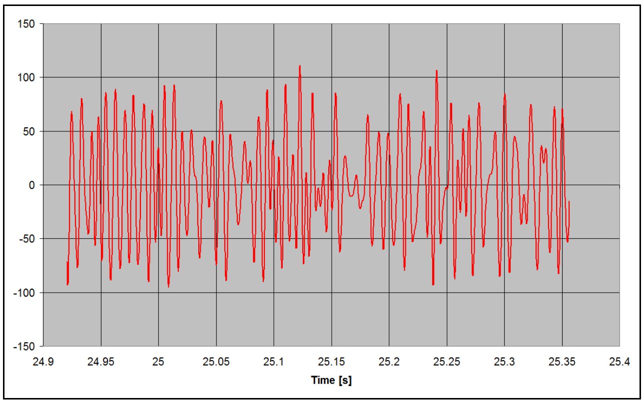


Figure 14.14. Force with rapidly variable behavior.

Once the calculation is finished, the analysis of the results is a bit more articulated than the frequency response and certainly more than a static one, mainly because we do not know where or when we have, for example, the maximum value of the Von Mises equivalent stress. Watching the animation of the time history helps to identify any response peaks at resonance and to focus around that instant of time analyzing the quantities of interest (stresses, displacements, reactions forces). Our example, again, is very simple and we expect to find the maximum values of all quantities around the maximum of the forces.

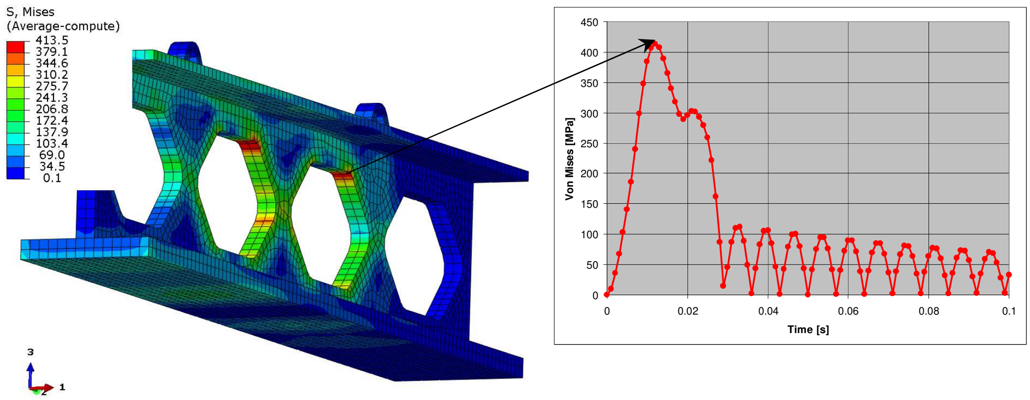


Figure 14.15. Contour of the Von Mises stress and time evolution of the same in the point of maximum.

And so it is: figure 14.15 shows the contour of the equivalent Von Mises stress at the instant of maximum value (0.012 s, i.e., 0.02 s later than when the forces reach their maximum) and the time trend of the Von Mises stress for the point where the maximum is located.

If we do the same thing for the vertical displacement of a point in the middle of the web at the free end we get the results in figure 14.16.

To summarize:

Minimum vertical displacement:	-7.46 mm
Maximum Von Mises stress:	414 MPa

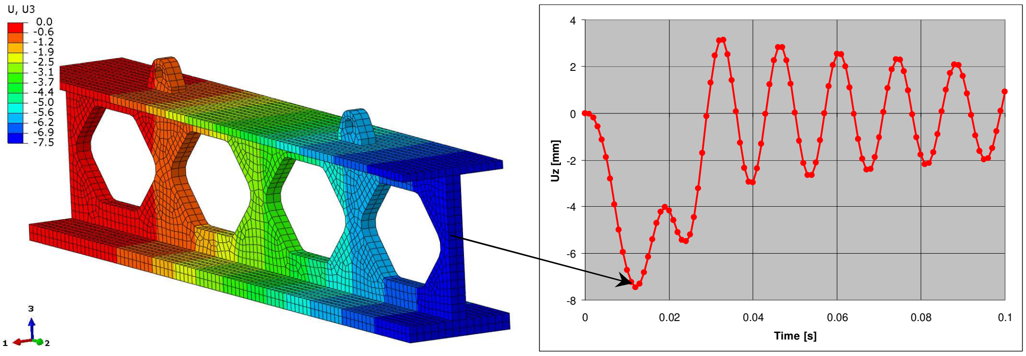


Figure 14.16. Contour dello spostamento in direzione verticale e andamento temporale dello stesso nel punto di minimo.

14.3.2 Modal superposition

As a first step we perform the modal analysis; since in the case of the beam clamped at both ends (see § 14.2) 10 modes were sufficient, we start by considering 10 modes. Table 14.4, similarly to 14.1, contains the corresponding effective modal masses and the percentage of the total mass in each direction. We observe that in the z-direction, where we expect the maximum response, the percentage is greater than the 85% given as the minimum value; we should therefore obtain a response very similar to that obtained with direct integration. Let's see. Also in the case of modal superposition we need to indicate a time step and also in this case are valid both the rule of thumb and the considerations expressed above. Let's choose a step equal to 0.001 s and proceed with the calculation. In figure 14.17 we report the time trends of the displacement and the equivalent Von Mises stress for the same nodes used in the case of direct integration and we superimpose them precisely on the latter for a more immediate comparison.

MODE	X-COMPONENT	Y-COMPONENT	Z-COMPONENT	X-ROTATION	Y-ROTATION	Z-ROTATION
1	4.507E-01	4.133E-28	1.238E-27	2.215E-21	1.665E+04	1.505E+06
2	3.542E-03	5.845E-28	2.939E-26	1.004E-19	2.227E+04	1.323E+04
3	7.255E-28	1.605E-05	5.170E-01	1.588E+06	3.232E+02	1.003E-02
4	1.223E-01	6.581E-29	1.655E-28	4.429E-22	6.693E+03	3.852E+04
5	6.255E-25	8.838E-05	1.191E-01	4.462E+03	7.445E+01	5.524E-02
6	1.017E-02	4.464E-27	5.188E-24	1.201E-19	2.864E+02	3.967E+03
7	1.002E-02	8.873E-28	3.578E-26	1.936E-21	3.815E+03	3.268E+03
8	1.079E-03	4.144E-28	2.247E-27	3.991E-21	4.416E+00	5.536E+02
9	5.669E-23	5.963E-04	2.535E-02	3.113E+03	1.584E+01	3.727E-01
10	1.348E-02	1.210E-22	2.640E-22	4.167E-17	3.496E+02	2.033E+03
TOTAL	6.113E-01	7.008E-04	6.615E-01	1.596E+06	5.048E+04	1.566E+06
%	82.5%	0.1%	89.3%	98.2%	82.4%	99.4%

Table 14.4. Effective modal masses for the first 10 modes. The sum in the z direction is 89% of the total mass.