14.6 Explicit methods

14.6.1 Introduction

Throughout the text the method used to solve the system of equations generated by the various models that we used in the examples is of implicit type.

However, since some years some FE codes of explicit type (we will see shortly what it means) have taken place, purposely studied for the simulation of strongly nonlinear dynamic and very short transient phenomena (that is they foresee contacts, extended and variable in time, plasticizations and failures), such as for example impact events. Through these codes it is possible to simulate shocks (as an example the impact of a bird with the leading edge of a wing or the crash of a car against a barrier), explosions (as an example of air bags) or some specific processes such as metal forming, presswork or machining in general.

The method has a very simple basis. It basically involves rewriting the general equation of motion

$$[M]\cdot\{\ddot{u}\}+[D]\cdot\{\dot{u}\}+[K]\cdot\{u\}=\{F(t)\} \qquad (14.1)$$

in the following way:

$$[M]\cdot\{\ddot{u}\}_t = (\{F\}-\{I\})_t \qquad (14.2)$$

being {F} the vector of external forces and {I} the vector of internal forces (elastic and viscous); in particular we have:

$$\{I\} = [D]\cdot\{\dot{u}\} + [K]\cdot\{u\} \qquad (14.3)$$

From (14.2) the acceleration can be easily derived:

$$\{\ddot{u}\}_t = [M]^{-1}\cdot(\{F\}-\{I\})_t \qquad (14.4)$$

If [M] is a diagonal matrix the calculation of its inverse does not present any difficulty and is very fast; [M] is diagonal if the finite elements used for the modeling of the mass use the method of the "lumped mass", that is, they concentrate the mass of the element in the nodes instead of distributing it over the entire domain of the element itself; this approximation does not alter the results in a sensitive way if the mesh is adequately dense.

Known the acceleration at the generic time t it is possible to derive by numerical integration (for example with Euler's method) the velocity at a subsequent instant:

$$\{\dot{u}\}_{t+\Delta t} = \{\dot{u}\}_t + \int_t \{\ddot{u}\}_t \cdot dt \qquad (14.5)$$

And similarly:

$$\{u\}_{t+\Delta t} = \{u\}_t + \int_t \{\dot{u}\}_t \cdot dt \qquad (14.6)$$

We can then proceed to calculate the acceleration at the time t+Δt and so on until the time t_{fin} in which we decide to stop studying the phenomenon.

The method is called "explicit" because in (14.4), (14.5), and (14.6) the unknown terms appear only to the left of the equal sign and therefore their determination is very fast; in fact, an explicit code takes only a fraction of the total solution time to carry out the explicit equations; most of the effort is in calculating the vector {I}, because it is still necessary to assemble the matrices [D] and [K] and to carry out the matrix products.

Clearly the method has some disadvantages; as an example it is not guaranteed the convergence of the solution and in order to reduce the risk to obtain wrong results the step of integration t must be assumed suitably small (some criteria exist to establish which is the maximum t usable). Secondly, it seems obvious looking at (14.4), that in order for the method to be applicable it is necessary to have accelerations in play and therefore it is not possible to study static phenomena, even if in reality with appropriate measures the method can be used satisfactorily also for "almost" static calculations; however, given the nature of the solution techniques adopted, a result will always be obtained, but there will be no real guarantee that it is "correct", something instead assured with an implicit method, where, if the convergence of the solution is reached, the results will be correct, at least within the limits of the quality of the model.

14.6.2 Comparison with the implicit method

Let's make a comparison with the beam used up to here, in particular the example in figure 14.12, solving the dynamic case with the explicit method. As mentioned, Δt must be chosen appropriately (we'll see some details later about the criteria on which the choice is based), and the current codes make the selection automatically, leaving the user to define Δt only for special cases. For our beam, therefore, we will only have to modify the type of solver to use and launch the calculation.

Figure 14.23 shows a comparison of the results in terms of Von Mises equivalent stress (for the same point in figure 14.15) and displacement (for the same point in figure 14.16). As we can see, the results are different, although definitely equivalent from an engineering point of view. We observe, however, that the deviation tends to increase as we approach the end of the analysis, and this can be explained by the "nature" of the explicit method we mentioned earlier: in particular, round-off errors add up and, consequently, analyses over long times are affected by larger errors. This explains why the explicit methods are very good for rapid phenomena and are not suitable, except for particular measures, for quasi-static phenomena.