possible crack, triggered by fatigue phenomena, will start, as we will see in Chapter 10. Clearly this type of plotting can only be applied to vector or tensor quantities.

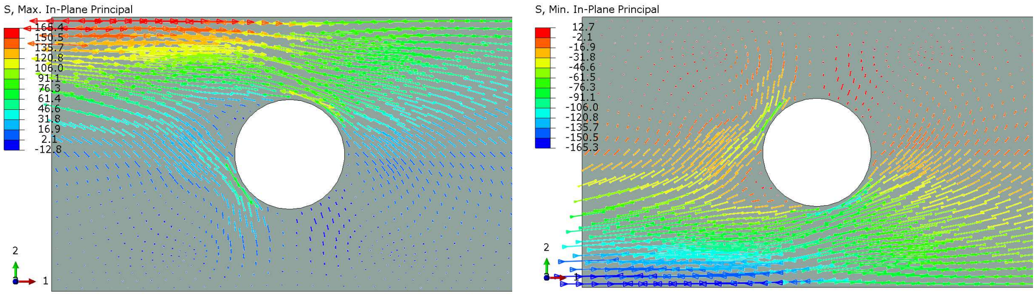

Figure 3.45. "Arrow contour" for the maximum and minimum principal stresses for the bending case (hidden mesh).

3.9.3 The funnel effect

Let's now take a look at an example where graphic post-processing can be misleading. We wish to calculate the rotation angle at the end of a tube subjected to the action of a torque. The manual calculation does not present any difficulty and Solid Mechanics gives us the following relation for the number sought:

$$\theta = \frac{M_t \cdot L}{G \cdot J_p}$$

being:
M_t the applied torque;
L the length of the tube;
G the shear modulus of elasticity for the tube material;
the polar moment of inertia of the tube.

In this case we are not interested in the values and their comparison with the theoretical case; we simply want to see what a graphical post-processor plots when a model reproducing this condition has been built.

Figure 3.46 illustrates the contour of the displacements in the circumferential and radial directions, respectively, of a cylindrical reference system (with origin on the axis of the tube) on the graphics of the non-amplified deformation. Beyond the numerical values (which, as we anticipated, are of no interest to us), the "funnel-shaped" opening effect is completely unrealistic and cannot be justified in any way from a structural point of view.

It is not a matter of mesh density or element type, aspects that, as we will see in Chapter 6, have an influence on the accuracy of the results. With a more or less fine mesh, or by employing solid elements, we would get the same qualitative results. Not only that; from the deformed shape in figure 3.46 we might be led to think that there is a radial component of displacement, but as we can see from the image on the right of the figure, it is zero, as it must be. So how do we justify the deformed shape in figure 3.46? The explanation is much simpler than it could be thought and resides in the fact that the calculation is linear: in this hypothesis deformed and undeformed shapes coincide, in the sense that it is supposed that the displacements are so small that they do not alter in a sensible way the geometry of the part. In our case this "linearization" translates into confusing the tangent with the corresponding arc; the post-processor, when it must plot the deformed shape of a structure, simply adds the corresponding displacement components to the coordinates of the nodes and then redraws the elements on the new coordinates thus obtained. In our example, since there are only circumferential displacement components, the nodes, being able to move only along the tangent because of what has just been said, will find themselves on a larger circumference, as it can be clearly seen by looking at figure 3.47.

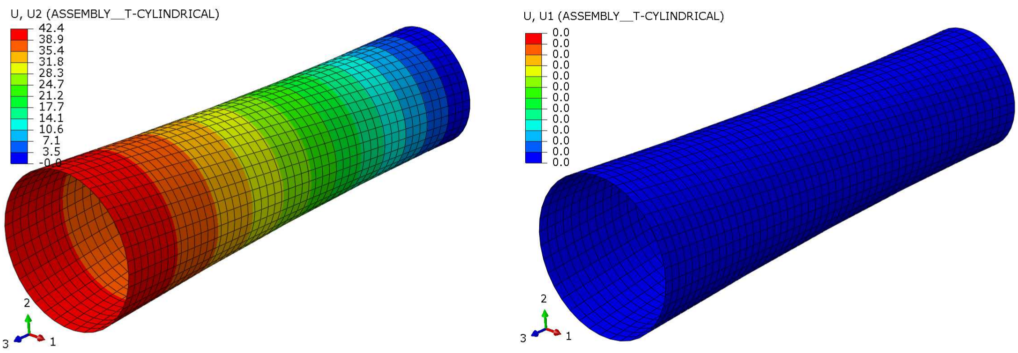

Figure 3.46. "Funnel" opening effect for a circular tube subjected to a torque: on the left the circumferential displacement, on the right the radial displacement.

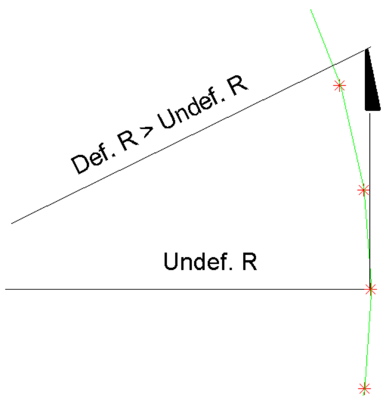

Figure 3.47. The circumference on which the nodes are located as a result of displacement, assuming small deformations, has a larger radius than the undeformed one.

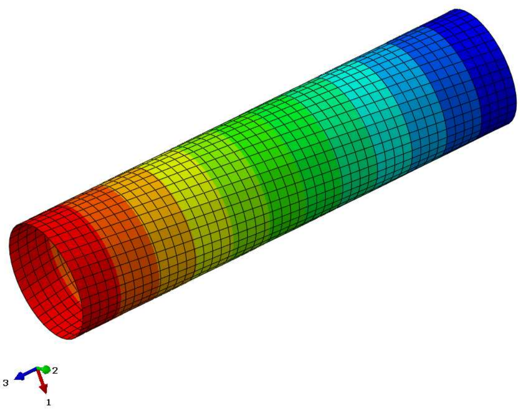

Figure 3.48. In a nonlinear analysis, the nodes follow the correct trajectory.

Obviously this effect is more and more marked as we move from the clamped base towards the extreme where the torque is applied, because the rotation θ, and therefore the tangential displacement, is proportional to the length; hence the "funnel" effect is justified.

This is surely an "aesthetic" problem; however, care must be taken because in less simple cases the plotting of a deformative behavior that has nothing to do with reality could be taken as valid. Especially today, when simple static graphics have been joined by animations that are so evocative that they are considered to contain no errors.

Finally, we show in figure 3.48 the deformation of the tube in the case in which a nonlinear analysis of geometric type has been conducted (large deformations, see Chapter 11); in this hypothesis the arc is no longer confused with the tangent and the nodes follow the trajectory they should.

3.9.4 Strain energy

A useful tool is the plotting of the "strain energy". In the past, as obviously it was for stresses, this quantity was simply tabulated element by element, while now the visual impact offered can be of great help to the structural engineer to quickly identify the areas of intervention, for example to stiffen or lighten a given structure.

As the term itself suggests, the strain energy is the elastic energy that an organ stores as a result of the deformation it undergoes due to the loads that stress it; clearly this energy is also equal to the work of the external forces. For an element whose stiffness matrix is [K] (see Appendix B), the strain energy is calculated as follows:

$$E = \frac{1}{2} \cdot \{u\}^T \cdot [K] \cdot \{u\}$$

being {u} the displacement vector of the nodes of the element in consideration. Once the finite element model is solved, the displacements of all the nodes of the structure are known and it is then possible to calculate the strain energy of each individual element. Needless to say, the sum of the energies of all elements must be equal to the energy calculated through the global stiffness matrix and the vector containing all displacements of all nodes. A plotting of the deformation energy allows, therefore, to see which are the areas that "work" more and which less; in essence through this information it is possible to determine where to add material to reduce stresses and deformations and where instead perhaps the structure can be lightened.

We then often refer to the strain energy density, which is the energy stored per unit volume. Figure 3.49 shows an example of total strain energy (left) and strain energy density (right) for a plate with a hole subjected to bending. Since the thickness here is constant, the two graphs provide the same information; in the case of more complex structures, where variations in thickness may occur, strain energy density is a better tool in evaluating where to operate most efficiently.