Figure 5.7 shows the finite element model used for the calculation, while figure 5.8 shows the deformed shape of first buckling mode. From the results file, we find that the corresponding buckling load is 268240 N, with an error of 0.2%.

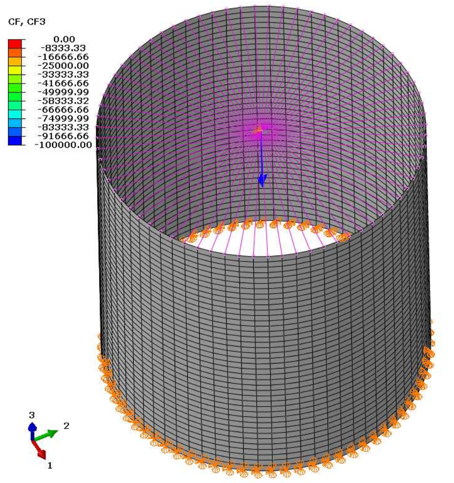

Figure 5.7. Finite element model for the axially loaded cylinder. The constraints are of hinge type. The load was applied at the center and distributed on the boundary through a rigid Multi Point Constraint (MPC) type element.

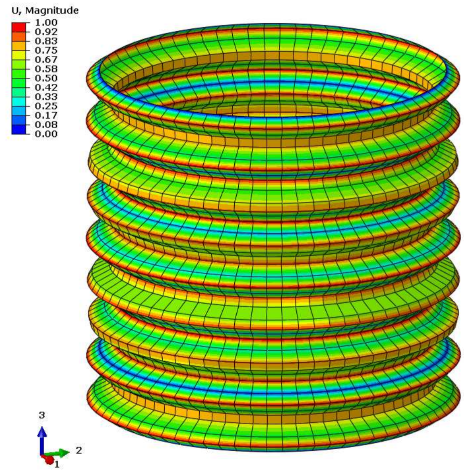

Figure 5.8. Deformed shape of the first buckling mode.

5.4.4 Thin-walled cylinder undergoing pure torsion

The last example concerns torsional buckling. As it is known in a pure torsion the principal stresses are equal in modulus and of opposite value. Therefore, in the direction in which the main compressive stress occurs (inclined by 45° with respect to the axis of the cylinder), a local buckling can be reached bringing the structure to overall collapse.

For this calculation we will use again the cylinder of the previous example and we will use the same model in which, clearly, we will replace the axial compressive load with a torque.

By following the procedure given in [3], we derive the value of the shear stress that generates the buckling for the cylinder with the geometric characteristics of our example:

$$\tau_{cr} = \frac{C \cdot \pi^2 \cdot E}{12 \cdot \left(1 - \nu^2\right)} \cdot \left(\frac{s}{L}\right)^2$$

where the coefficient C is obtained from the graph of the figure 5.9 (that interpolates some experimental values) as a function of the value of

$$Z_L = \frac{L^2}{R \cdot s} \cdot \sqrt{1-\nu^2} = 1192$$

It is derived that C = 169 and therefore τ_{cr} = 43 MPa.

Now, using Bredt's formula, we are able to calculate the value of the torque that generates the torsional buckling.

$$\tau_{cr} = \frac{M_{tcr}}{2 \cdot \Omega \cdot s}$$

Being $\Omega = \pi \cdot R^2$ we get:

M_{tcr} = 13508848 Nmm.

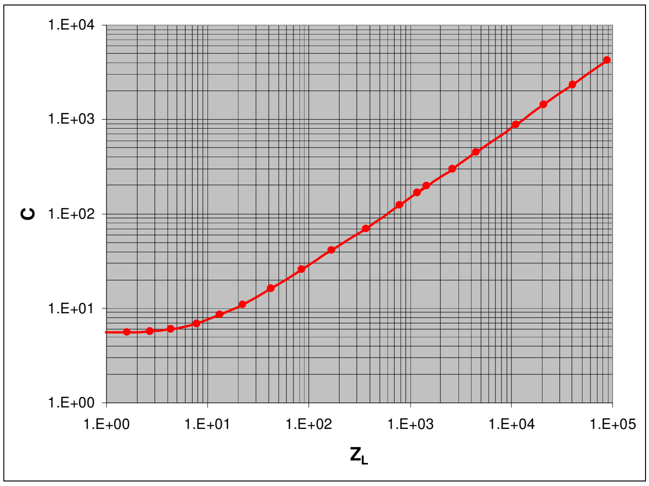

Figure 5.9. C-Z_L diagram.

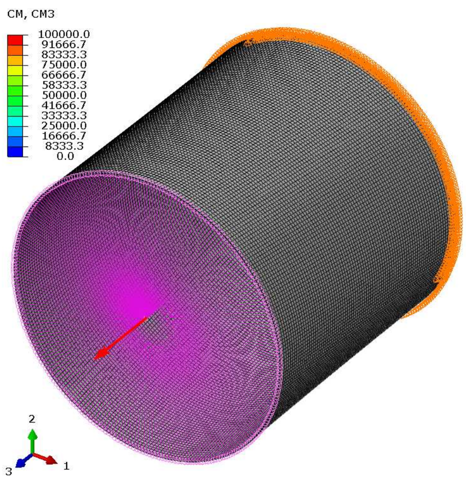

Figure 5.10. The cylinder in figure 5.7 is now loaded in torsion and the mesh has been refined.

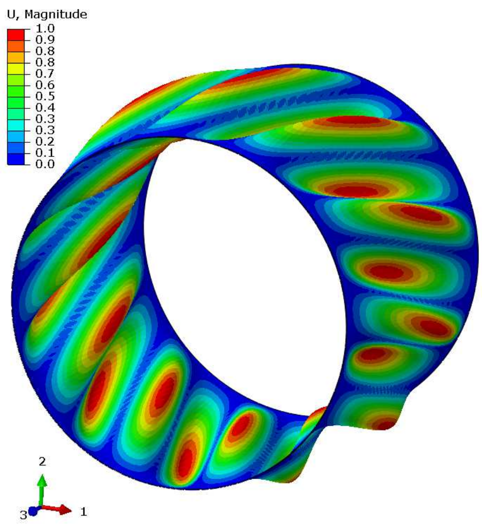

Figure 5.11. First buckling mode for the cylinder undergoing torsion.

Figure 5.10 shows the model; compared to that of figure 5.7, the mesh has been refined to avoid an overestimation of the stiffness (see Chapter 6) and we have replaced the axial force with the torque; figure 5.11 shows the first buckling mode of the cylinder in discussion. The critical torque value is equal to 14884000 Nmm, with an error of just over 9%. However, given the nature of the C coefficient, it cannot be ruled out that the agreement is actually much better.

5.5 Notes on instability in nonlinear domain

If the limitations posed by the linearity assumption are too restrictive (for example, because it is suspected that, under the action of loads, the material may exceed its elastic limit at some point in the structure), it is necessary to use a non-linear calculation. In this case it is possible to take into account all forms of non-linearity, presented later in the text in Chapters 11, 12 and 13. For example, for the cylinder subjected to external pressure, taking into account the follower effect (i.e. the forces that "follow" the structure during its deformation) for the pressure load would allow a better agreement with the theoretical results.

Clearly the non-linear calculation is the one that comes closest to reality. Not only that, in this way it is also possible to evaluate what happens in a structure when only one part of it has buckled; obviously it doesn't make sense to adopt this procedure on a single beam subjected to an axial load of compression, because in this case it would collapse. However there are cases in which a part of the structure becomes unstable, without the whole collapsing catastrophically. For example, the plates of thin-shell ribbed structures can reach instability: in this case the plates are no longer able to withstand the load, but if this has a way to flow through other parts of the structure (the ribs) then there is no collapse of the whole. This is where the post-buckling analysis becomes interesting, which basically represents the calculation of how loads are redistributed within the part of the structure which has remained stable.

In Chapter 11 we will examine some examples of post-buckling structures.