CHAPTER 6
Errors in Finite Element calculation

6.1 Introduction

So far we have seen how to "handle" a finite element model and how to handle the results it produces. However, these numbers are largely influenced by the way the model is constructed and the numerical quality with which the equations that the model generates are solved.

It is clear that the "trash in-trash out" principle applies, i.e., if the input contains errors, the output cannot be expected to be correct.

The errors to which a numerical analysis is submitted are many: they range from gross errors of the user (for example wrong description of the material) to those related to the software used for the construction of the model, from the errors of the method (which, we recall, approximates the solution) to errors of numerical nature.

In the next few paragraphs of this Chapter we will address all of these types of errors, highlighting the dangers, where possible, through simple illustrative examples.

6.2 User errors

This type of error is perhaps the most difficult to address because it has to do with the uniqueness of each individual and it is therefore unpredictable.

However there are some classic cases, related to the necessity to introduce numbers in the model; today in fact we are used to work with graphical systems and the necessary keyboard inputs are very few. So, sometimes, it can happen to type simple numbers incorrectly; for example it can happen to insert the wrong value of Young's modulus for the material that constitutes the structure to be calculated; it can happen to insert wrong values of forces and/or pressures, temperatures and so on.

A more conceptual error, but still related to numerical input, is the one based on measurement units: calculation codes should all be "unitless", i.e. the consistency of the adopted system should be the responsibility of the user. If, for example, the International System (SI) is used, there is no mistake, but it is required that linear dimensions be expressed in meters, and this is sometimes a problem (especially in the mechanical field) because of the small numbers (think of node coordinates) involved. Often then it is common to use "hybrid" systems in order to manage easily the geometry of the model, but it is necessary to pay attention to the types of analysis for which this model is used. A classic example is the use of the "modified" SI, using millimeters instead of meters; the other quantities involved are the forces (in Newton), the masses (in kilograms), the time (in seconds). For static analysis there are usually no problems: Young's modulus is expressed in MPa, as the pressures, the density of materials is introduced in kilograms per cubic millimeter. Apparently everything works, because even the total mass (which the code calculates as density times total volume) gives a correct value. However if the same model is used also for a dynamic analysis (without getting to complicated things it is sufficient to think about the calculation of eigenfrequencies) here the results obtained are "scaled" by a certain factor. Let's see why. Sticking to the example of the calculation of eigenmodes, the relationship that gives the oscillation frequency of a mass-spring system with mass M and stiffness K is:

$$\omega = \sqrt{\frac{K}{M}}$$

Using SI we would have that the dimensions of K and M would be [N/m] and [kg] respectively. Switching to basic units we will have (remember that $1\ \mathrm{N} = \frac{1\ \mathrm{kg \cdot m}}{\mathrm{s}^2}$):

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{\left[\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}\right] \cdot \left[\frac{1}{\mathrm{m}}\right]}{[\mathrm{kg}]}} = \left[\frac{1}{\mathrm{s}}\right]$$

With the modified SI, on the other hand, the dimensions of K are [N/mm] and thus:

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{\left[\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}\right] \cdot \left[\frac{1}{\mathrm{mm}}\right]}{[\mathrm{kg}]}} = \sqrt{\frac{\left[\frac{\mathrm{kg \cdot 1000mm}}{\mathrm{s}^2}\right] \cdot \left[\frac{1}{\mathrm{mm}}\right]}{[\mathrm{kg}]}} = 10 \cdot \sqrt{10} \left[\frac{1}{\mathrm{s}}\right]$$

So the calculated frequency is falsified with respect to the actual frequency (in particular it is lower than the actual frequency). To put things right it is sufficient to express the masses in tons (i.e. the densities in tons per cubic millimeter). In fact:

$$\omega = \sqrt{\frac{K}{M}} = \sqrt{\frac{\left[\frac{\mathrm{kg \cdot m}}{\mathrm{s}^2}\right] \cdot \left[\frac{1}{\mathrm{mm}}\right]}{[\mathrm{t}]}} = \sqrt{\frac{\left[\frac{\mathrm{kg \cdot 1000mm}}{\mathrm{s}^2}\right] \cdot \left[\frac{1}{\mathrm{mm}}\right]}{1000[\mathrm{kg}]}} = \left[\frac{1}{\mathrm{s}}\right]$$

In this case it is necessary to remember that the total mass provided by the model will be in tons and not in kilograms. Not only that, if by chance the model was subjected, for the static analysis, to inertial loads (those that arise from masses and accelerations), such as weight force and/or centrifugal force, the corresponding gravitational and centripetal accelerations will be expressed not in meters per second squared but in millimeters per second squared.

In an attempt to prevent the inexperienced user from making these errors (which can be very serious), many of the more recent calculation codes have introduced the use of measurement units. The user can then choose between various systems of homogeneous units. Very often in fact we can find images with the stresses full scale in mMPa (yes, just milli-mega-Pascal: too simple to indicate kPa, kilo-Pascal!); and this because a method to overcome the drawback seen is to express the forces in mN (milli-Newton).

Luckily for the experts, for now these calculation codes have not yet completely taken over from the user; it is still possible therefore to "force" them to use the numbers that everyone prefers, assuming the responsibility of the choice.

Another "keyboard input " error is the wrong assignment of the thickness value to shell elements or to plane stress elements; to avoid these errors the only system is to carefully recheck the model and, always a good rule, try already to have an idea of the order of magnitude of the expected results (for example the stress value), maybe with first approximation manual calculations.

Clearly the user can also make mistakes in the "mouse inputs" and therefore assign forces, constraints, thermal loads, pressures to the wrong nodes and elements; even in these cases the main control system is a careful investigation of the model, except for some tricks that we will see in Chapter 9.

6.3 Discretization errors

6.3.1 Introduction

Discretization error is intrinsic to the FEM. In fact, it has been said (and further details can be found in Appendix B) that the FEM provides an approximate solution; the less coarse the discretization, the more accurate will be the results produced. We emphasize that here with discretization we mean the way in which a structure is approximated: we refer to the type and number of elements that we decide to use for a given model.

6.3.2 Mesh density

The number of elements is qualitatively referred to as "mesh density". It must be said that in general finite element modeling tends to overestimate the actual value of the structure's stiffness, and this is more true the less fine the mesh is. This fact results in the underestimation of the stress state; let's see why.

6.3.2.1 A borderline case

We know that a rod subjected to an axial force F undergoes a deformation ε given by:

$$\varepsilon = \frac{\Delta L}{L}$$

being L the initial length of the rod; we also know that $\sigma = \varepsilon \cdot E$.